Education 147-*Stela Anguelova*

UNC Math 22 Section 003 - Calculus for Social Sciences
(Formerly known as Business Calculus)

Unit - Applications of the Derivative (Chapter 4)

Placement: A score of 520 or higher on the SAT II, Math, Level I or Math, Level II C exam, a score of 2 on the Calculus AP exam (or BC sub-score), or a passing grade in Math 10 is required. Considerable facility with algebra and word problems is vital for success in this class. A student who has received credit for Math 31 is not allowed to take this course. With such a broad background requirements, this course could be diverse and include students with diverse majors in their first, second, third and fourth year of studies.

In the current section I teach there are 8 first year students, 6 second year students, 7 juniors, 5 seniors and even 1 graduate student.

Course Description: Math 22 is a survey of differential and integral calculus of one variable. It is a terminal course and will not prepare for Math 32. (If you plan to apply to UNC's business school, you may have to take OR 22, not Math 22. Check with your advisor.) Student mastery of pre-calculus algebra skills is assumed from the start. If student's algebra skills are rusty, he/she may find it helpful to review some algebra on his/her own. I have provided my students with an algebra self-test on the first day of class and I have asked them to complete it as homework to identify possible algebra troubles early on. Since word problems requiring calculus techniques for their solution form a large portion of the material, clear and correct mathematical communication of students' thinking is a must.

Rationale

This unit further explores the power of derivatives as a tool to help analyze the properties of functions. The students would see how the techniques known can be used to obtain information about functions and then how this information can be used to accurately sketch graphs of those functions.

Students would also see how the derivatives could be appropriately used in solving a large class of optimization problems that are applicable in various situations, including finding what levels of production will yield a maximum profit for a company, finding what level of production will result in minimal cost to a company, finding the maximum height attained by a rocket, finding the maximum velocity at which air is expelled when a person coughs, and various other problems applicable in virtually any field of life.

It is essential that in the course of presentation students become well aware of the use of technology. Students should learn and master all the algebraic techniques, but they should also be encouraged to make use of it and particularly their calculators and make sure their algebraically obtained answers make sense.

This unit addresses the need for all UNC students in all fields of studies to have at least some initial exposure to basic differential analysis techniques. By including diverse application problems in the lesson presentation and homework exercise sets, and particularly ones taken from current relevant events, all students are put in a better disposition to relate and thus find the theory more worthy and become motivated to learn it.

Section Objectives for Students • Math 022
text: Tan, *Calculus for the Managerial, Life, and Social Sciences*, 7th ed.

This checklist is posted in Blackboard to help my students keep focused and know exactly what they need to know by section. I also use it to compile a list concatenating those for each section and chapter and give it out as review guides for tests

Section 4.1 Applications of the First Derivative

terms, formulas, and theorem statements to know:

- increasing on open interval, decreasing on open interval
- relative minimum, relative maximum, relative extremum
- (first-order) critical number
- relative extreme (minimum, maximum) values of a function
- first derivative test for continuous functions

concepts:

- understand that the first derivative test may fail if the function has discontinuities

procedures:

- use 1st derivative to determine open intervals on which function is increasing/decreasing
- determine (1st-order) critical numbers
- use 1st derivative test to find relative maxima and relative minima of a **continuous** function

Section 4.2 Applications of the Second Derivative

terms, formulas, and theorem statements to know:

- concave up on open interval, concave down on open interval
- (second-order) critical number
- inflection point
- relative extreme (minimum, maximum) values of a function
- second derivative test for relative extrema of twice-differentiable functions

informal reminders:

- 2nd derivative positive on interval, concave up on interval
- 2nd derivative negative on interval, concave down on interval
- with addition of horizontal tangent line (chin indentation or mustache),

Minnie and Maxxie for the 2^{nd} derivative test for relative extrema 

concepts:

- understand the limitations of the second derivative test for relative extrema
- understand that at an inflection point, two things happen: 1^{st}) there is a tangent line (so first derivative is defined there or else tangent line there is vertical) and 2^{nd}) the concavity changes at that point
- understand what the sign of the second derivative says about the behavior of the first derivative
  (particularly in context of word problems)
- understand what existence of an inflection point says about the behavior of the first derivative
  (particularly in context of word problems)

procedures:

- use 2^{nd} derivative to determine open intervals on which function is concave up/ concave down
- determine (2nd-order) critical numbers
- use 2^{nd} derivative to find inflection points of a differentiable function

Section 4.3 Curve-Sketching

terms, formulas, and theorem statements to know:

- horizontal asymptote (verbal descriptions and limit definitions)
- vertical asymptote (verbal descriptions and limit definitions)

procedures:

- find asymptotes of a rational function
- use the curve-sketching guide on p. 290

major errors to avoid:

- thinking that the graph of a continuous function cannot intersect a horizontal asymptote; some do, some don't

Section 4.4 Optimization I

terms, formulas, and theorem statements to know:

- absolute maximum value, absolute minimum value
- theorem 3

procedures:

- find absolute extrema of a continuous function on a closed, bounded interval
- use the curve-sketching guide on p. 290

Section 4.5 Optimization II

formulas to memorize or be able to derive:

- for a rectangle, Area = (length)(width) and Perimeter = 2(length) + 2(width)
- for a closed box (right rectangular cylinder) , Volume = (length)(width)(height) and Surface Area = 2(length)(width)+2(length)(height)+2(width)(length);
- derive similar formulas for an open box (missing at least one side)
- for a closed can (right circular cylinder), Volume = $\pi r^2 h$ and Surface Area = $2\pi rh + 2\pi r^2$;
- derive similar formulas for a can open at one or both ends
- for a circle, Area = πr^2 and Circumference = $2\pi r$

other useful formulas:

- for a sphere, Surface Area = $2\pi r^2$ and Volume = $4\pi r^3/3$

procedures:

- use guidelines in box on p. 315

omitted:

inventory control

Processes

Students would develop the following processes (organized by section)

4.1
use 1st derivative to determine open intervals on which function is increasing/decreasing
determine (1st-order) critical numbers
use 1st derivative test to find relative maxima and relative minima of a continuous function

4.2
use 2nd derivative to determine open intervals on which function is concave up/ concave down
determine (2nd-order) critical numbers
use 2nd derivative to find inflection points of a differentiable function

4.3
find asymptotes of a rational function
use the curve-sketching guide on p. 290

4.4
Employ derivatives to find absolute extrema of a continuous function on a closed, bounded interval

4.5
Modeling an application problem and then use derivatives to find absolute extrema of a continuous function on a closed, bounded interval

Anguelova TR Math 022 Schedule - Fall 05

Please note the position of this unit in the time-frame of the entire semester calendar. The assessment for this unit (Test 2) is disjoint from the presentation of the unit on purpose. I always like to give students a couple of days to reflect and ask me questions before they are tested on anything.

DAY	SECTION	TOPIC
8/30	Intro, 2.4	Limits
9/1	2.5	One-sided limits & continuity
9/6	2.6	**last day to add**; the derivative
9/8	3.1, 3.2	basic differentiation rules, product & quotient rules
9/13	3.3	chain rule
9/15	3.3,3.4	same, marginal functions in economics
9/20	3.5,3.7	higher-order derivatives, differentials
9/22	3.7, catch-up	differentials
9/27	Test 1	
9/29	4.1	**1st-yr Early Warnings Due 10/3**; applications of the 1st derivative
10/4	4.2	applications of the 2nd derivative
10/6	4.3	**drop day 10/10**; curve-sketching
10/11	4.4, 4,5	optimization I, optimization II
10/13	4.5	optimization II
10/18	5.1, 5.2	exponential functions, logarithmic functions
10/25	Test 2	
10/27	5.4	differentiation of exponential functions
11/1	5.5	differentiation of logarithmic functions
11/3	5.6	exponential functions as math' models
11/8	6.1	antiderivatives & rules of integration
11/10	6.2	integration by substitution
11/15	6.3,6.4	area & the definite integral, Fundamental Theorem of Calculus
11/17	6.5	evaluating definite integrals
11/22	Test 3	
11/29	6.6	area between two curves
12/1	7.4	**last possible test day**; improper integrals
12/6	7.5	applications of calculus to probability
12/8	review, eval's	
12/13	**FINAL**	**Tuesday, 4:00 – 7:00 PM**

9/29	4.1	applications of the 1st derivative first derivative test increasing/decreasing understand the concept of critical points understand the notion of extremum (see the parallel between graphical and algebraic way to find it) understand when a first derivative test fails
10/4	4.2	applications of the 2nd derivative concave upward/ concave downward find second order critical points concept of inflection points relative extrema second derivative test understand when the second derivative test fails
10/6	4.3	Quiz curve-sketching concept: horizontal asymptote concept : vertical asymptote find asymptotes of rational function sketching guide graph can intersect HA but never VA

10/11	4.4	optimization I concept: absolute maximum/minimum find absolute extrema curve-sketching
10/13	4.5	Quiz optimization II modeling solve application problems curve-sketching
10/25	Test 2	Administer Test 2 in class
11/03	Test 2 Follow-up	Go over Test 2 problems Stress the explanations on the major common mistakes and write the complete solutions for all problems Solution Key is then placed outside my office for reference

Math 022 -- Fall 2005 -- Practice Problem List for this Unit
(*Calculus for the Managerial, Life, and Social Sciences*, Tan, 7^{th} edition)

4.1	2(inc' on (1, ∞),constant on (-1,1),dec' on (– ∞, -1)),5,15,25,31,33,43,44(ans: c.),45,46 (ans: b.),51,61,65,77,85,99
4.2	5,7,9,11,14 (rate greatest at 10, inc' between 8 and 10, dec' between 10 and 12),15,21,27,33,45,49,53,55,59,61,63,75bc,81
4.3	5,9, odds 21-29, 35,37,41,43;53,57, sketch graph of C(x)=.5x / (100-x), then 61
4.4	1,3,5,12 (abs' min' is 0;occurs at (0,0); no abs' max),13,21,25,33,47,51,61,67
4.5	3,7,11,12(radius (18/Pi)^(1/3), height 2*(18/Pi)^(1/3)),17,21,25 and on p. 91f. 72 (A=40x-x^2,for 0≤ x≤40),74 (V= x(8-2x)(15-2x) for 0<x<4),75,78 (A=52-2x-50/x for x>0),79

This unit is designed for my Calculus class. Each lesson would take 1hr and 15 minutes - the entire course period at UNC.

My students are familiar with basic notions such as functions, continuity, differentiability, limits. They are also proficient with the various rules of differentiation. We have just completed a section on applications of the first derivative and various marginal functions in Economics and other sciences. In this unit I am supposed to introduce some powerful techniques to analyze functions more efficiently. Once the background is established we would be getting into optimization problems. Immediately after this unit we would use these techniques and apply them on logarithmic and exponential based models. After that we would get into the second large topic for this course- Integral Calculus.

Section 4.1 Applications of the First Derivative

I. Anticipatory Set (7 min)

I want you to recall problem 6 from the test review session in which I asked you to find the marginal average cost function, given the cost function C(x)=5000+2x
We found the AC=C/x=5000/x+2, from which we found MAC=$(5000/x+2)' = -5000/x^2$
Can we interpret what this means? … (brief class discussion)
The purpose of this section is to be able to tackle problems like these.

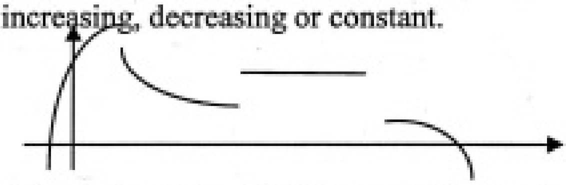
II. Objective and Lesson Purpose (3 min)

We would learn the following procedures (which turn out to be quite useful in modeling):

- use 1st derivative to determine open intervals on which function is increasing/decreasing
- determine (1st-order) critical numbers
- use 1st derivative test to find relative maxima and relative minima of a **continuous** function

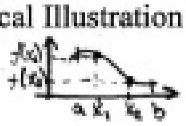
III. Instructional Input and Modeling (20 min)

Background:
Example 1 (going up/down & higher/lower language is ok here)
Given the graph of the function bellow determine the intervals on which the function is increasing, decreasing or constant.



Easy! What about if I did not give you the graph, but gave you a complicated formula for a function?

Graphical Illustration   Definition (Increasing/Decreasing Functions)



A function f is decreasing on an interval (a,b) if for any two numbers x_1 and x_2 in (a,b), $f(x_1)>f(x_2)$, whenever $x_1< x_2$.

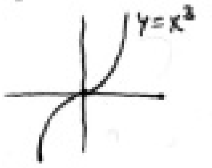
A function f is increasing on an interval (a,b) if for any two numbers x_1 and x_2 in (a,b), $f(x_1)<f(x_2)$, whenever $x_1< x_2$.

Recall The derivative, i.e. the slope of the tangent line to the graph of a function, gives and idea about the shape of the graph of that function.

Theorem 1

i) If $f'(x)>0$, for every x in (a,b), then f increases on (a,b)
ii) If $f'(x)<0$, for every x in (a,b), then f decreases on (a,b)
iii) If $f'(x)>0$, for every x in (a,b), then f is constant on (a,b)

Graphical Illustration



Example 2

Given $y=x^3$, find where is y increasing, decreasing or contant.
Solution : $y'=3x^2>0$ for all x, so therefore by Theorem 1 y is an increasing function for all x, i.e. for $x \in (-\infty, \infty)$. This is also seen from the graph of y, which we are familiar with.

Remark If f' is continuous, it cannot change sign unless it equals 0 somewhere. So

Procedure for finding the intervals on which f is decreasing or increasing

1. Find all x for which $f'(x)=0$ or f' is discontinuous and identify the open intervals determined by these numbers
2. Select a test point c in each interval found in 1. and determine the sign of $f'(c)$ on that interval
3. If $f'(c)>0$, f increases on the interval
   If $f'(c)<0$, f decreases on the interval

IV. Guided Practice-Checking for Understanding (30 min)

Problem (students work in 5 groups of 4 and present solutions in front of class)
Determine where are the functions increasing or decreasing.
(Do those algebraically, but you can check your answers using your graphic calculators).

1. $f(x)=x^3+9x^2-21x+50$
2. $g(x)=x^{4/7}$
3. $s(t)=\dfrac{2t}{t^2+1}$
4. $g(x)=x\sqrt{x+1}$
5. $h(x)=\dfrac{x^2}{x-1}$

   Remark We must not automatically conclude that h' changes sign when we move across a number where h' is discontinuous or a zero of h'.

Note f' can help locate high and low points on the graph. Show a stock graph online and copy to board to show the slopes in another chalk color and discuss increasing and decreasing around extrema.

Definition

A function f has a relative maximum at x=c if there exists an open interval (a,b) containing c such that $f(x) \le f(c)$ for all x in (a,b).
A function f has a relative minimum at x=c if there exists an open interval (a,b) containing c such that $f(x) \ge f(c)$ for all x in (a,b).
Relative maximum and relative minimum are called extrema (singular is extremum)

Remark. Slopes in each case extrema with signs chart

Definition

A critical number of a function f is any number x in the domain of f such that $f'(x)=0$ or $f'(x)$ does not exist.

The First Derivative Test
1. Determine the critical numbers of f 2. Determine the sign of $f'(x)$ to the left and right of each i)If f' changes sign from − to + as we move across the critical number c, then f(c) is a relative minimum. ii)If f' changes sign from + to − as we move across the critical number c, then f(c) is a relative maximum. iii)If f' does not change sign as we move across the critical number c, then f (c) is not a relative extremum.

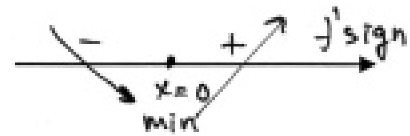
Example 3

Find the relative extrema, if any to the following functions

1. $f(x)=x^{2/3}+2$
2. $g(x)=\dfrac{x}{x^2-1}$

Solution:

1. $f'=\dfrac{2}{3(x)^{1/3}}$, *which is discontinuous at =0. The sign chart is*



So there is a relative minimum at (0,f(0))=(0,2)

2. $g'=\dfrac{1+x^2}{(x^2-1)^2}$ *Since g' is never 0, x=1 and x=-1 are not critical points (they are not in the domain of the function). Therefore g has no relative extrema.*

Example 4 (Applied)

According to a study conducted in 1997, the revenue (in millions of dollars) on the US cellular phone market in the next 6 years is approximated by $R(t)=0.03056t^3-0.45357t^2+4.8111+31.6$ (t is measure in years and is between 0 and 6 with t=0 corresponding to 1997).

a. Find the intervals on which the revenue increases/decreases
b. What does the result tell you about the revenue in the cellular market in the years under consideration?

Solution

a. $R'=0.09168t^2-0.90714t+4.81111$ *is a polynomial, so it is continuous everywhere.*

$R'=0$ *yields* $t=\dfrac{0.90714\pm\sqrt{0.90714^2-4(0.09168)(4.81111)}}{2(0.09168)}$. *Since D=-1.76<0,*

R appears to have no critical points. R'(0)=4.81111>0. So R increases everywhere on (0,6).

b. The result in a shows that the revenue on the cellular market increases throughout those 6 years under consideration.

V. Independent Practice (13 min)

Problem 2

Let f(x)= $\begin{cases} -3x, x<0 \\ 2x+4, x\geq 0 \end{cases}$

a) f'(x)=? Show it changes from negative to positive
b) Show f has no relative minimum at x=0. Does this contradict with the first derivative test? Explain

---

Solution (Posted on Blackboard website a day before next class)

a) f'(-1)=-3
f'(1)=2, so f' goes from negative to positive across x=0

b) No f does not have a relative minimum at x=0, because f(0)=4 but f(x)<4 if x is a little less than 4. This last statement does not contradict the first derivative test because f' is not continuous at x=0.

Homework 2(inc' on (1, ∞),constant on (-1,1),dec' on (– ∞, -1)),5,15,25,31,33,43,44(ans: c.),45,46 (ans: b.),51,61,65,77,85,99

VI. Closure (2 min) So why do you think the first derivative is a powerful tool?

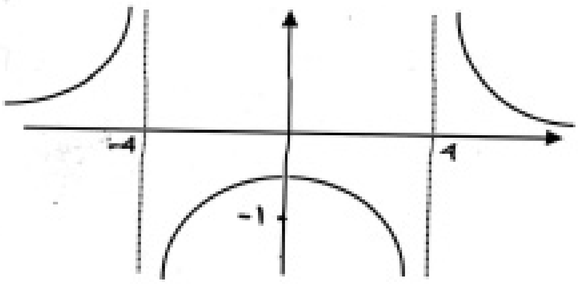
The first derivative is a powerful tool at analyzing function. It gives us an insight as to when functions change behavior and where extrema are to be expected. This is useful in any science.

Section 4.2 Applications of the Second Derivative

I. Anticipatory Set (5 min)

Recall The derivative, i.e. the slope of the tangent line to the graph of a function, gives an idea about the shape of the graph of that function.

Example 1 I want you to recall problem 5 from the previous section in which you were given the following graph.



What can you tell me about the slopes to this graph?
Where do the slopes increase? Decrease?
Plot the slopes in colored chalk as students discuss the behavior of the slopes on the intervals $(-\infty, -4)$, $(4, \infty)$ and $(-4, 4)$

Anything else you observe about this graph? Dips up/down. Discuss informally.

II. Objective and Lesson Purpose (3 min)

We would learn the following a more formal analysis procedure to determine the shape of a function on intervals (which turns out to be quite useful in modeling):

- use 2nd derivative to determine the intervals of concavity of a function
- determine (2nd-order) critical numbers and inflection points
- use 2nd derivative test to find relative maxima and relative minima of a **continuous** function

III. Instructional Input and Modeling (35 min)

Definition (Concave Up/Concave Down Functions on an Interval)

For a differentiable function on (a,b)
A function f is concave upward/concave up on an interval (a,b) iff f' increases on (a,b).

A function f is concave downward/concave down on an interval (a,b) iff f' decreases on (a,b).

Where does the graph lie in comparison to tangent lines?
Remark:
Concave up: graph lies entirely above tangent lines
Concave down: graph lies entirely below tangent lines

Definition (Concave Up/Concave Down Functions at a Point)

A function f is concave upward/concave up at x=c if there exists an interval (a,b) containing c where f is concave up/upward on (a,b). A function f is concave downward/concave down at x=c if there exists an interval (a,b) containing c where f is concave down/downward on (a,b).

Recall: f'' measures the rate of change of f'. So
f''>0 implies slopes of tangent lines increase
f''<0 implies slopes of tangent lines decrease

Theorem

i) If $f''(x)>0$, for every x in (a,b), then f is concave up on (a,b)
ii) If $f''(x)<0$, for every x in (a,b), then f is concave down on (a,b)

Procedure for determining the intervals of concavity

1. Find all x for which $f''(x)=0$ or f'' is not defined to identify the open intervals determined by these numbers
2. Select a test point c in each interval found in 1. and determine the sign of $f''(c)$ on that interval
3. If $f''(c)>0$, f is concave up on the interval
   If $f''(c)<0$, f concave down on the interval

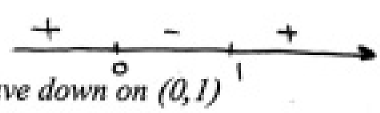
Remark Choose the representative points wisely to make your calculations as simple as possible and thus to minimize the chances to make a mistake.

Example 2
$f(x)=x^4-2x^3+6$

Solution

$f'(x)=4x^3-6x^2$
$f''(x)=12x^3-12x=12x(x-1)$

f'' is continuous everywhere so the chart is:



Thus f is concave up on $(-\infty,0)\cup(1,\infty)$ and concave down on (0,1)

Don't erase the calculations to this example yet will need it to illustrate inflection points a bit later.

Remark Please plot this *function on your calculators now and check the answers. They* should match

IV. Guided Practice-Checking for Understanding (25 min)

Problem (students work independently and present solutions in front of class)
Determine where the following functions are concave up and concave down.
(Do those algebraically, but you can check your answers using your graphic calculators).

1. $g(x)=x^{1/3}$ ($g'(x)=\frac{1}{3}x^{-2/3}$, $g''(x)=-\frac{2}{9}x^{-5/3}$ + (0) − & g'' not defined @ $x=0$)
2. $s(t)=\frac{t}{t^2+1}$ ($s'(t)=\frac{1-t^2}{(1+t^2)^2}$, $s''(t)=\frac{-2x(3-x^2)}{(1+x^2)^3}$ − ($-\sqrt{3}$) + (0) − ($\sqrt{3}$) + ...)
3. $g(x)=-\sqrt{4-x^2}$ ($g'(x)=\frac{x}{\sqrt{4-x^2}}$, $g''(x)=\frac{4}{\sqrt{(4-x^2)^3}}$ >0 whenever g is defined...)

Definition A point on the graph of a continuous function f, where the tangent line exists and where concavity changes is called an inflection point.

> Procedure for Finding Inflection Points
> 1. Compute f''(x)
> 2. Determine the numbers, c's , in the domain of f for which f''(x)=0 or f''(x) does not exist
> 3. Determine the sign of f'' on the left and right of each point found in 2. If a change of sign is observed moving across x=c, (c, f (c)) is an inflection point.

Back to Example 2 the inflection points are *(0,6) and (1,5)*

Problem (students work independently and present solutions in front of class)
Determine the inflection points of the following functions if those exist
(Do those algebraically, but you can check your answers using your graphic calculators).

4. $g(x)=(x-2)^{4/3}$ ($g'(x)=\frac{4}{3}(x-2)^{1/3}$, $g''(x)=\frac{4}{9(x-2)^{2/3}}$, $x=2$ is a candidate but $g''(x)>0$ for all x so it fails to be an inflection point)

Example 3
Given that f(1)=2, and that f'(x)>0 on $(-\infty,1)\cup(1,\infty)$, and that f''(1)=0, determine the correct graph of f a) b) c)

(Ans: (b))

Example 4The graphs below were used by a police commissioner to illustrate what effect a budget cut would have on crime in a city. The number N1(t) gives the projected number of drug-related crimes in the next 12 months.N2(t) gives the projected number of drug-related crimes in the same time-frame if the next year's budget is cut.

a) explain why n1'(t) and N2'(t) are both positive on (0,12)
b) What are the signs of N1''(t) and N2''(t) on (0,12)?
c) Interpret your results in b).

Solution

a) N1'(t)>0 and N2'(t)>0, so but N1 and N2 are increasing on (0,12) *b) N1''(t)<0 on (0,12) and N2''(t)>0 on (0,12)*

c) Although the projected number of drug-related crimes will increase in either case a cut in the budget will see an accelerated increase in the number of crimes committed. With the budget intact the rate of increase of crimes committed will continue to drop.

The Second Derivative Test
1. Compute f'(x) and f''(x)
2. Find all critical points of f at which f'(x) =0
3. Compute f'' (c) for each such number
a) If f'' (c) <0, (c, f (c)) is a relative maximum
b) If f'' (c) >0, (c, f (c)) is a relative minimum
c) If f'' (c) =0, the test is inconclusive, i.e. it fails to determine the extremum

Remark Look at the chart at the bottom of page 275 it is a helpful graphical representation which could prove helpful

Problem (students work independently and present solutions in front of class)
Find the type of relative extrema for the following functions
(Do those algebraically, but you can check your answers using your graphic calculators).

5. $f(x)=2x^2+3x+7$ ($f'(x)=4x+3 \Rightarrow CP: x=-\frac{3}{4}$; $f''(x)=4>0 \Rightarrow g''(-\frac{3}{4})>0$ so $(-\frac{3}{4}, \frac{47}{8})$ is rel. min)

6. $g(x)=\dfrac{2x}{x^2+1}$ ($g'(x)=\frac{2(1-x)(1+x)}{(x^2+1)^2} \to x=\pm 1$; $g''(x)=\frac{4x(x^2-3)}{(x^2+1)^3}$, $f''(-1)=1>0$ so $(-1,-1)$ is rel. min; $f''(1)=\frac{4(-2)}{8}<0$ so $(1,1)$ is rel. max)

7. $g(x)=x^2+2/x$ ($g'(x)=2x-\frac{2}{x^2} \to x=1$; $g''(x)=2+\frac{4}{x^3}$, $g''(1)=6>0$ so $(1,3)$ is a rel. min)

Example 4 (Applied)

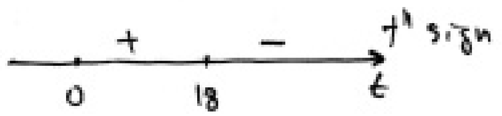
The altitude in feet of a rocket t seconds into flight is given by
$S=f(t)=-t^3+54t^2+480t+6$, where t is non-negative

Find the point of inflection of f and interpret your result. What is the maximum velocity attained by the rocket?

Solution

$V=f'(t)=-3t^2+108t+48$

$A=f''(t)=-6t+108=-6(t-18)$



The point (18,f(18)=20310) is the inflection point of f. The maximum velocity of the rocket is attained when t=18. It is f'(18)=1452ft/sec

V. Independent Practice (0 min)

Homework 5,7,9,11,14 (rate greatest at 10, inc' between 8 and 10, dec' between 10 and 12),15,21,27,33,45,49,53,55,59,61,63,75bc,81

Closure (2 min) So why compute the second derivative?

The second derivative is a powerful tool at analyzing function. It gives us an insight as to when rates of functions change behavior and what types of extrema are to be expected. This is useful in any science.

Quiz 5 (on 4.1 and 4.2) ***(12 min)***

Given $f(x)=x^4-2x^2$, find

1. the intervals on which the function is increasing/decreasing,
2. the relative extrema (if any).
3. the intervals on which the function is concave up/down
4. the inflection points

Be sure to give me all the details as to how you obtained your answers.

Section 4.3 Curve Sketching

I. Anticipatory Set (3 min)

I noticed on the quiz you just finished the majority of you were using your graphic calculators and trying to graph the function I gave you to analyze.

Why did you do that? …

The graph of a function is a very useful tool for visualizing the behavior of that function. It is nice to have it. So today we would learn how to sketch the graph of a function even when we do not have our graphic calculators at our disposal.

II. Objective and Lesson Purpose (3 min)

Recall The derivative, i.e. the slope of the tangent line to the graph of a function, gives an idea about the shape of the graph of that function.

Prerequisites: We know how to determine where a function is increasing, decreasing, concave up and concave down. We also know how to find relative minima/maxima and inflection points.

Today we would use those along with horizontal and vertical asymptotes to help us graph functions.

III. Instructional Input and Modeling (25 min)

Definition Vertical Asymptote (VA)

The line x=a is a vertical asymptote of the graph of a function f if either

$$\lim_{x\to a^+} f(x)=\infty \text{ or } \lim_{x\to a^+} f(x)=-\infty$$

$$\lim_{x\to a^-} f(x)=\infty \text{ or } \lim_{x\to a^-} f(x)=-\infty$$

Remark The vertical asymptotes are not part of the graph, but they are helpful aid in sketching graphs. This is why we use dashed lines.

Recall For rational functions.

$f(x)=P(x)/Q(x)$, (where both $P(x)$ and $Q(x)$ are polynomials)

the line $x=a$ is a vertical asymptote of f if $Q(a)=0$ but $P(a)$ is different from 0.

Definition Horizontal Asymptote (HA)

The line y=b is a horizontal asymptote of the graph of a function f if either

$$\lim_{x \to \infty} f(x) = b \text{ or } \lim_{x \to -\infty} f(x) = b$$

Remark Polynomials have no vertical or horizontal asymptotes

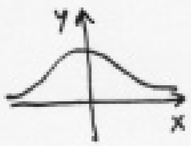
Remark HAs can be intersected but VAs cannot be. Why? (Domain…)

Remark Both HAs and VAs are lines and should be written in line equation form (e.g. y=c, x=a)

Example 1

Find the horizontal and vertical asymptotes of the following function (if any)

$$y = \frac{1}{1+x^2}$$



Solution

VA: Trying to *solve $x^2+1=0$ to find VAs. Since it has no solutions, there are no vertical asymptotes*

HA: $\lim_{x \to +\infty} y = 0$ *and* $\lim_{x \to -\infty} y = 0$ *So y=0 is a horizontal asymptote*

Ask students to graph this function on their calculators and discuss briefly how we see the horizontal and vertical asymptotes from the graph. Show that the calculations make sense and that the graph represents what happens algebraically.

Example 2

$$y = \frac{x^2-4}{x^2-1}$$

Solution

VA: $x^2-1=(x-1)(x+1)=0$ has solutions x=1 and x=-1

$$\lim_{x \to -2^+} y = \infty \quad \lim_{x \to -2^-} y = -\infty$$

$$\lim_{x \to 2^+} y = -\infty \quad \lim_{x \to 2^-} y = \infty$$

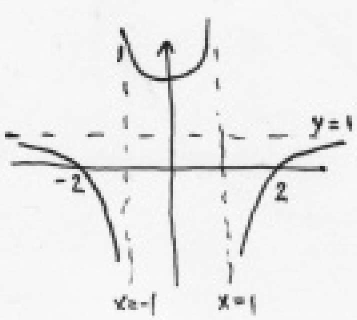
So VAs: x=-1 and x=1

HA: $\lim_{x \to +\infty} y = \dfrac{x^2(1-\frac{4}{x^2})}{x^2(1-\frac{1}{x^2})} = 1$

So horizontal asymptote at y=1. Does the function graph intersect horizontal asymptote?

Check $\dfrac{x^2-4}{x^2-1}$ *=1 for solutions. There are none*

$x^2-4=x^2-1$ yields -4=-1 There *are no solutions so graph does not intersect HA*



IV. Guided Practice-Checking for Understanding (20 min)

Problems (students work- I ask volunteers to come up to the board and do those. I guide through the analysis if needed and make sure the details are well written).
Find the vertical and horizontal asymptotes of the following functions:

1. $f(x)=1+\dfrac{2}{x-3}$ (Ans: VA : x=3, HA: f=1)
2. $g(x)=\dfrac{2-x^2}{x^2+x}$ (Ans: VA: x=0 and x=-1, HA: g=-1)
3. $h(x)=\dfrac{x^4-x^2}{x(x-1)(x+2)}$ (Ans: VA: x=-2, HA: None)

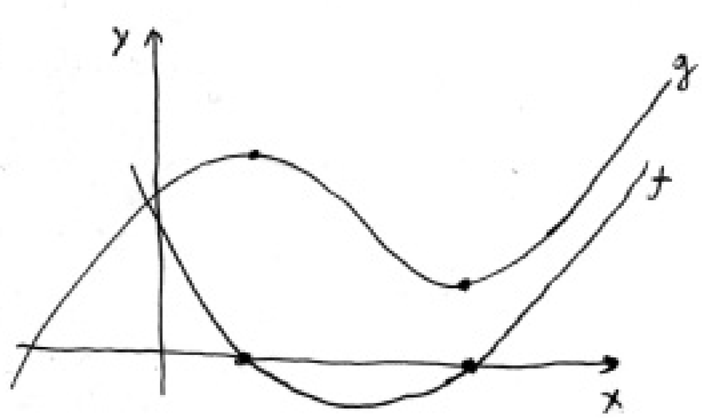
   Remark Note that h(x) has holes at x=0 and x=1, not vertical asymptotes. This should be well stressed because I have noticed that students find the distinction between holes and vertical asymptotes hard to grasp.

Guide to Curve Sketching (f)

1. Determine the domain of f
2. Find the x- and y- intercepts of f (Note that if f(x)=0 is too difficult to solve, we do not find the y-intercept)
3. Determine the behavior of f for large in absolute value x's((limits as x goes to + and – infinity)
4. Find the HAs an VAs (if any)
5. Determine the intervals on which f increases/decreases
6. Find the relative extrema of f (if any)
7. Determine the intervals of concavity
8. Find the inflection points (if any)
9. Plot additional points if needed to identify the shape of the graph and then sketch it.

Example 3 (Intuition Example)
Given the following graphs of the functions f and g can you determine which is the derivative of the other one?

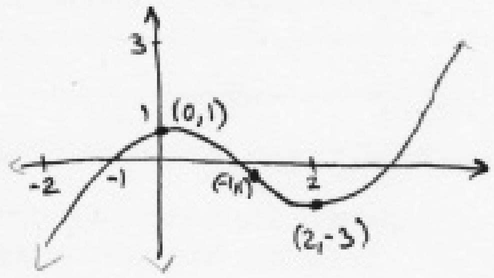
What is your logic, please explain? (brief class discussion here)



Summary
Notice that f is one degree lower than g (g looks like 3rd degree polynomial, while f looks like a quadratic)
Also notice that at the relative min/max of g, f(x)=0.

Example 4
Given $f(x) = x^3-3x^2+1$
Sketch its graph

Solution
$D=(-\infty, \infty)$
$f(0)=1$, $x^3-3x^2+1=0$, too complicated to solve
Polynomial so there are no HAs and VAs
Increasing on $(-\infty,0) \cup (2,\infty)$
Decreasing on (0,2)
Minimum (2,-3), Maximum(0,1)
Concave up on $(1,\infty)$
Concave down on $(-\infty,1)$
Inflection point (1,-1)
We use those to obtain the graph on the right



Is the graph consistent with what we know about polynomial graphs of third degree with positive coefficient in front of highest order term?

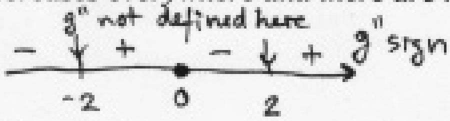
Example 5
Given $g(t)= \dfrac{t}{t^2-4} = \dfrac{t}{(t-2)(t+2)}$

Sketch its graph

Solution
D: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$
Intercept(0,0)
VA: $t=2, t=-2$
HA: $y=0$

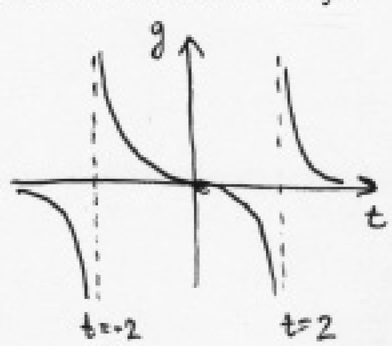
$g' = \dfrac{t^2-4}{(t^2-4)^2} < 0$, *so g decreases everywhere and there are no extrema*

$g'' = \dfrac{2t(t^2+12)}{(t^2-4)^3}$



So concave up on $(-2,0) \cup (2,\infty)$ *and concave down on* $(-\infty,-2) \cup (0,2)$

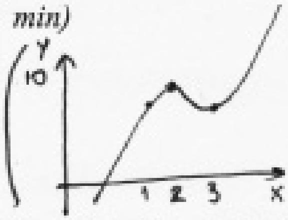
Is the graph consistent with what we know about graphs of rational functions? Why?

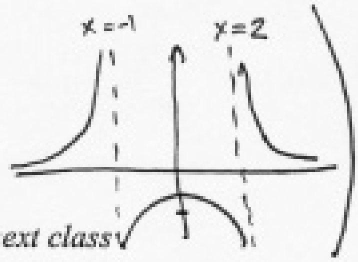


V. Independent Practice (10 min)

Problem 2 Sketch the graph of

1. $f(x)=2x^3-15x^2+36x-20$
2. $g(t)=\dfrac{1}{t^2-t-2}$





Solutions to these are posted on Blackboard website a day before next class

Homework 4.3/ 5,9, odds 21-29, 35,37,41,43,53,57, sketch graph of C(x)=.5x / (100-x), then 61

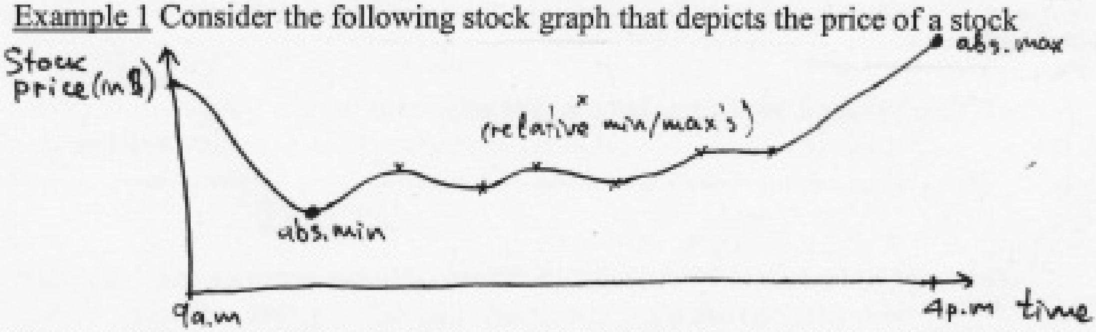
VI. Closure (2 min)

The graph of a function is a very powerful tool for visualizing the behavior of a function. Based on just a few carefully chosen conditions, we are able to create a full analysis of that function graph it and then use the graph to obtain further information about the function not yet algebraically established.

Section 4.4 Optimization I

I. *Anticipatory Set (5 min)*

Example 1 Consider the following stock graph that depicts the price of a stock



Can you tell me what is the highest price at which it traded for yesterday? What about the lowest price? How do you know?

The lowest and highest points on a graph of a function give is the absolute extrema of the function.

II. *Objective and Lesson Purpose (3 min)*

Often times in life we need to know the minimum and maximum of a function or how to achieve it. This is what optimization is about and it is what we would be concerned with for the rest of this unit.

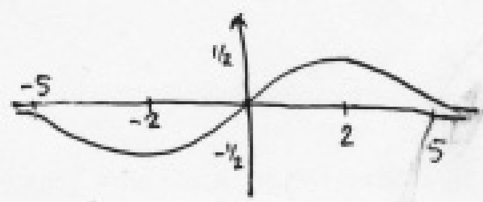
III. *Instructional Input and Modeling (22 min)*

Definition The Absolute Extrema of a function f

If $f(x) \leq f(c)$, for all x in the domain of f, then $f(c)$ is called the absolute minimum value of f

If $f(x) \geq f(c)$), for all x in the domain of f, then $f(c)$ is called the absolute maximum value of f

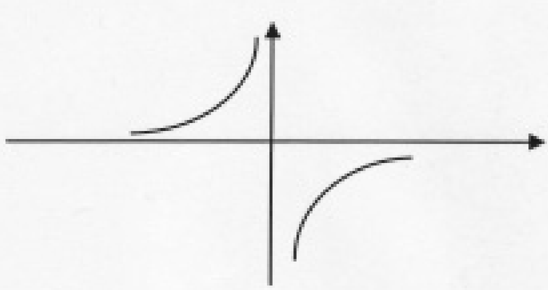
Example 2 Given the graph below identify the absolute min/max of the function

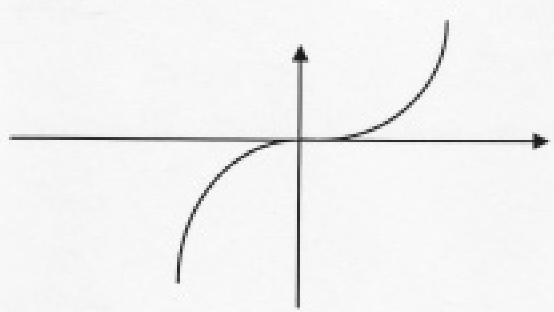


abs min : (-2,-1/2)
abs max: (2, 1/2)

Remark

The following depicted functions have no absolute maxima or minima





Theorem 3
If a function f is continuous on a closed interval [a,b], then f has both an absolute maxumum and an absolute minimum on [a,b].

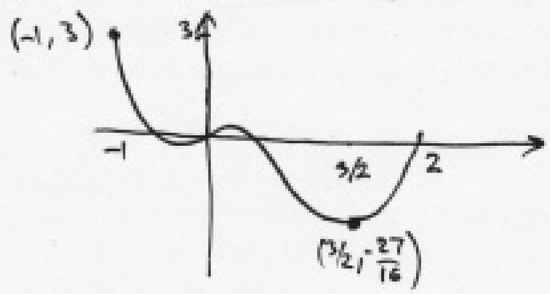
Remark Absolute extremum of a continuous function f occurs on (a,b) if a relative extremum of f occurs on (a,b), i.e.corresponding x's are critical points.
Remark For a closed interval extrema can be achieved at the endpoints endpoints.

Back to stock graph illustration briefly discuss the relative and absolute extrema and illustrate the remarks above.

Finding the Absolute Extrema of a Function on a Closed Interval

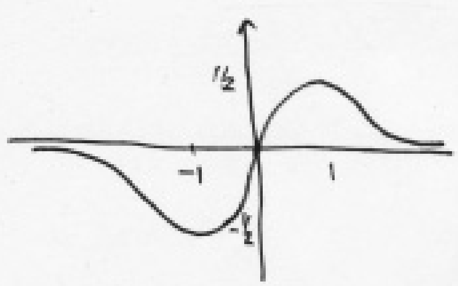
1. Find the critical points of f on (a,b)
2. Compute f at each critical point found in step 1 and compute f(a) and f(b)
3. The absolute max/min correspond to the largest/smallest numbers found in step 2.

Example 3 Find the absolute min/max of the functions if those exist



abs min : (3/2 , -27/16)
abs max : (-1 , 3)

Example 4 Find the absolute min/max of the functions $f(x)=x/(1+x^2)$ if those exist



abs min : (1 , 1/2)
abs max : (-1 ,-1/2)

Example 5 Find the absolute extrema of

1. $f(x)=x^4/2-2x^3/3-2x^2+3$ on [-2,3]

This is a polynomial and thus continuous and differentiable on [-2,3]

CPs: $f'=2x^3-2x^2-4x=2x(x-2)(x+1)=0$ which gives x=-1, x=0 and x=2

Next compute:

x	-2	-1	0	2	3
f (x)	25/3	13/6	3	-7/3	15/2

So the absolute maximum of f is f (-2)=25/3 and the absolute minimum is f (2)=-7/3

I. *Guided Practice-Checking for Understanding (40 min)*

Problem (students work independently and present solutions in front of class)

Find the absolute extrema of the following functions on the given interval

1. $s(t)=\frac{t}{t-1}$ on [2,4] ($s'(t)=-\frac{1}{(t-1)^2}$, no critical points; t=1 not in domain of s: $s(2)=2$ abs. max, $s(4)=4/3$ abs. min

t	2	4
s(t)	2	4/3

)
2. $f(x)=9x-1/x$ on [1,3] ($f'=9+\frac{1}{x^2}=0$ has no real solutions: $f(1)=8$ abs. min, $f(3)=\frac{80}{3}$ abs. max)
3. $f(t)=t^{2/3}(t^2-4)$ on [-1,3] ($f'=\frac{8(t^2-1)}{3t^{1/3}}=0 \Rightarrow t=\pm1$, undefined @ t=0

t	-1	0	1	3
f(t)	-3	0	-3	$5\cdot3^{2/3}$

abs min @ (-1,-3) & (1,-3). Abs max @ $(3,5\sqrt[3]{9})$ Rmk: Absolute extrema need not be unique)

Application Problem 1- **Maximizing Profit**

The weekly demand for Sony plasma TV is given by P (x)=-0.05x+600 ($0 \le x \le 12000$), where p is the wholesale unit price in dollars and x is the quantity demanded. The weekly total cost function associated with manufacturing there television sets is $C(x)=0.000002x^3-0.03x^2+400x+80000$, which is the cost incurred in producing x units. Find the level of production that would yield the maximal profit.

Solution $R(x)=px=-0.05x^2+600x$

$P(x)=R(x)-C(x)=-0.000002x^3-0.02x^2+200x-80000.$

To maximize P on [0,12000] differentiate it and set it equal to 0:

$P'=-0.000006x^2-0.04x+200=0$ *gives*

$3x^2+20000x-100000000=0$

x=

$$\frac{-20000\pm\sqrt{20000^2+1200000000}}{6}=-10000 \text{ or } x=3{,}333.3.$$

x=3,333.3 is a critical point on [0,12000] but to make it feasible we round it to the nearest whole number 3,333.

So the table is

x	0	3,333	12,000
P (x)	-80,000	290,370	-4,016,000

Application Problem 2- **Office Rents**

After the economy softened, the sky-high office space rents of the late 1990s started to come down to earth. P gives the approximate price per square foot in dollars. Let R (t) be the prime space in Boston Back Bay and Financial district form 1997(t=0) to 2002 and let $R(t) = -0.711t^3+3.76t^2+0.2t+36.5$, where $(0 \le t \le 5)$.

Show office space rents peaks at about the middle of 2000. What was the highest office space rent during the given period?

Solution

Optimizing R (t):

$R'(t) = -2.133t^2+7.52t+0.2=0$ gives t approximately equal to -0.026 or 3.55. Since –0.056 is not in the given interval [0,5], the optimal value is about 3.5 years. We are not certain whether this is a maximum or a minimum, but finding *$R''(3.5) = -4.233(3.5)+7.52<0$ implies that we have indeed found the maximum. So* office space rents peaks at about the middle of 2000, as suggested.

The highest office space rent is given by $R(3.55)=52.79$, so $52.79 /sq ft is the highest office space rent within the period 1997-2002.

II. *Independent Practice (0 min)*

Homework

4.4/ 1,3,5,12 (abs' min' is 0;occurs at (0,0); no abs' max),13,21,25,33,47,51,61,67

III. Closure (5 min)

Optimization proves necessary in many aspects of life. This is why studying the techniques to find extrema is essential. Using calculus techniques to optimize functions and particularly differentiation proves to be quite efficient at finding algebraically when and what extrema functions are achieved.

Name______________________________
Math 22-Anguelova

Quiz 6

Given the function $g(x)=\dfrac{1}{x^2-x-2}$

a. Find the domain of g and any intercepts it has

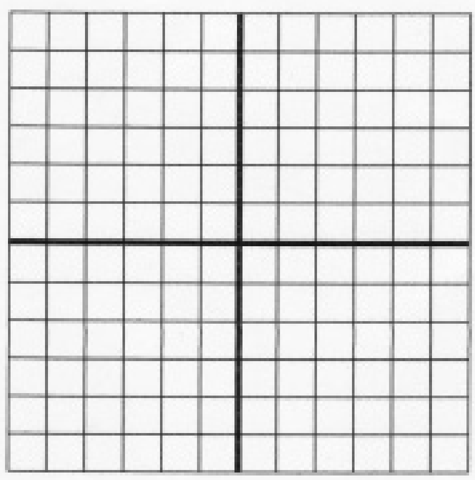
b. Find its asymptotes (give your answers in equation form)

c. Find the intervals on which g increases, decreases along with any extrema it has

d. Find the intervals of concavity and any inflection points g has.

3)a)
b)
c)
d)

e) Use all the above parts to help you graph g (you are free to rescale those axes any way you find fitting, but be sure to label)



Section 4.5 Optimization II

I. Anticipatory Set (2 min)

What did optimization involve? Ask students to recall

Find the largest/smallest…..

Recall Last time we optimized application problems in which we were given specific functions.

Now what if we were not given a function explicitly, but we had some other information or data given? How would this change our optimization task?

II. Objective and Lesson Purpose (1 min)

In this section we would optimize functions, which we would construct, modeling a problem given

Particularly helpful formulas would be the ones in the handout provided including the formulas for perimeter, area and volume of all the basic shapes (distribute handout to students).

III. Instructional Input and Modeling (10 min)

Guideline for Solving Optimization Problems

1. Assign a letter for each variable mentioned in the problem. If appropriate draw and label the figure.
2. Find an expression for the quantity to be optimized
3. Use the conditions given in the problem to write the quantity to be optimized as a function of one variable. Note there could be restrictions to be placed on the domain of f from the physical considerations of the problem
4. Optimize the function f over the domain using the methods of 4.4

How do we conduct 4? Briefly prompt students to recall and voice out the techniques.
Remark What are we looking for when optimizing a continuous function on a closed interval?…What are the candidates?…. We look among the critical points and the end points of the interval values of that function.

Remark Now what if the domain is not closed?….. We resolve to a graphic method

Application Problem 1

Say I want to optimize the planting area in a greenhouse, which I am about to build. Suppose the greenhouse I want is 6.5feet tall and I have $160ft^2$ of insulating material at my disposal. Assuming that I will use all the insulation material at my disposal and that my greenhouse has a flat roof what would be the optimal dimensions of the greenhouse?

Solution

On one side the planting area is A=lw. Unfortunately this is a functions of two variables. I only know how to optimize functions in one variable. But luckily I have not used all the information given, maybe I can bypass that.

I know all sides of the greenhouse and the roof need to be isolated with the 160 ft^2,material so that is 2 long sides, two short sides and the roof:

$160=2(6.5)w+2(6.5)l+lw$.

Therefore $w=(160-13l)/(13+l)$.

Substituting this in the area function to be optimized yields

$A=(160l-13l^2)/(13+l)$

This is a function of one variable which I know how to optimize.

Can anyone give me a suggestion what to do?

I take the derivative and set it equal to 0

$A'=[(160-26l)(13+l)-160l-13l^2)]/(13+l)^2=0$

This implies that $160.13-26.l-l^2=0$.

This is a quadratic equation with $D=132+160=18.14$.

Thus there would be two solutions. However only the positive solution makes sense, for a length. So $l=-13+18.14=5.14ft$.

The problem asked for both dimensions so I go back and substitute the length to find the width:

$w=(160-13.(5.14))/(13+5.14)=5.14ft$

Note that $l=w=5.14ft$, *which makes sense since it is consistent with our knowledge that a square gives the maximal area.*

IV. Guided Practice-Checking for Understanding (50 min)

Problems

Students work in groups of 5 on the following 4 problems and then are required to present their solutions in front of the class. Each groups works on a different problem for about 20 minutes and then is given 7 minutes to present their solution.

Problem1 **Fencing a Garden**

A man wishes to have a rectangular-shaped garden in his backyard. He has 50ft of fencing material with which to enclose his garden. Find the dimensions for the largest garden he can have if he uses all the fencing material

Problem 2 **Packaging I**

By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, the cardboard may be turned into an open box. If the cardboard is 16 inches long and 10 inches wide, find the dimensions of the box that would yield the maximum volume.

Problem 3 **Optimal Subway Fare**

A city's Metropolitan Transit Authority(MTA) operates a subway line for commuters from a certain suburb to the downtown metropolitan area. Currently an average of 6000 passengers a day take the trains, paying a fare of $3.00 per ride. The board of the MTA, contemplating raising the to $3.50 per ride in order to generate a larger revenue, engages the services of a consulting firm. The firm's study reveals that for each $0.50 increase in fare, the rideship will be reduced by an average of 1000 passengers a day. Thus the consulting firm recommends that MTA stick to the current fare, which already yields a maximum revenue. Show that the consultants are correct.

Problem 4 **Packaging II**

Betty Moore Company requires that its corner beef hash containers have a capacity of 54 cubic inches, have the shape of right circular cylinders, and be made out of aluminum. Determine the radius and height of the container that requires the least amount of metal.

V. *Independent Practice (0 min)*

Homework
4.5/ 3,7,11,12(radius (18/Pi)^(1/3), height 2*(18/Pi)^(1/3)),17,21,25 and on p. 91f. 72 (A=40x-x^2,for 0≤ x≤40),74 (V= x(8-2x)(15-2x) for 0<x<4),75,78 (A=52-2x-50/x for x>0),79

VI. *Closure (2 min)*

Modeling functions and then using calculus to optimize them proves to useful in any field of life.
Next time we would see even more application problems, but this time involving logarithmic and exponential functions. We would still that the techniques we studied work for those functions as well.

Assessment

I particularly believe in informal when evaluating my students. I closely pay attention to the student involvement during the various activities I provide during class because class participation is a vital part of my assessment. I pay particular attention to individual and group activities performance, as well as my one-on-one interaction with my students during office hours, personal scheduled sessions and review sessions. I watch for correct strategies, appropriate language and terminology, as well as the eagerness to carry out the details. I keep a journal of my class participation impression notes and take it into account when writing my final grades for the course for each student. I can give a student a maximum of 7 % if I consider it appropriate based on my informal assessment.

I also give out regular (about once a week) short quizzes that are a minimal part of the students' final grade but they do give important feedback and practice before tackling my formal form of assessment which are the tests I give on every unit. To relieve some of the pressure off the quizzes I drop the two worst quizzes in the end of the semester and count the rest towards the maximum of 5% of my Math 22 final course grade.

I use the very same strategy for Homework problem sets. My students are given a list of problems for each section, which they collect and turn in to me every Thursday in the beginning of class. Each section is graded separately, which lets me keep track and identify which sections each particular student has problems on and gives me the chance to address the particular difficulties a student has. In the end of the semester I drop the two worst section grades and calculate the average for the rest to a maximum of 5% of the course grade. On sections with more than 10 problems due, I grade a carefully chosen representative selection among the problems. However I always post a complete detailed solution set outside my office for student to reference to.

The test included in this unit is worth 15% of the final course grade for my Math 22 students. It is a maximum of 100 points (plus 5 bonus points for each bonus question that is correctly completed). The points for each problem are indicated on the test. I do not expect each student to be able to tackle the bonus problems, but I always include those because I want to identify independent thinking and give credit for it. I also want to give extra points to my students who enjoy the challenge but are otherwise a little bored and get the algebra wrong on easier problems. I also do not want them to finish the test too early and be a shocking disturbance factor for the rest of my class. My bonus questions are typically very different from all problems my students have seen in lectures and homework exercises, but they are absolutely doable with the material covered with some further exploration.

Math 22

Section 3
Fall 2005
Exam 2

Name:______________________________________

Instructions:

- Show all your work on the test. Answers without adequate supporting work will receive **no** credit. Work may include explanations in sentence form. You may use the back of pages if needed.
- Simplify all answers.
- Misuse of mathematical notation will be penalized, as will failure to make complete mathematical statements.
- Keep answers exact (in terms of π, roots, fractions, etc.), unless otherwise noted.
- Calculators are not allowed.
- Box your final answers.
- Sign the Honor Pledge once you have completed the entire test.
- Your test should have 7 consecutively numbered problems and two bonus problems. Check now; see me immediately if your test is incomplete. You will receive no credit for problems not attempted because of an incomplete test.

I have neither given nor received any unauthorized help on this test, and I have conducted myself within the guidelines of the University Honor Code.

Pledge:______________________________________

Do your best … good luck!!

1.) (10 points) For the function $g(t)=12t^5+125t^3-13t+5$ list all the vertical and horizontal asymptotes it has(if any). Be sure to give me your reasoning.

1.) HA:
VA:

2.) (12 points) Given that the consumer demand for widgets is $q = 1800 - 45p$ cases of widgets when p is the price, in dollars, of a case of widgets, find the following:

a. A formula for the revenue, R(p). R(p) =

---

b. Find what price per case of widgets yields a maximal revenue (Be sure to show this is a relative maximum and not a relative minimum).

2)
a)

b)

3.) (20 points) Given the function $g(x)= \dfrac{x}{x^2-25}$

a. Find the domain of g and any intercepts it has

b. Find its asymptotes(give your answers in equation form)

c. Find the intervals on which g increases, decreases along with any extrema it has

3)a)

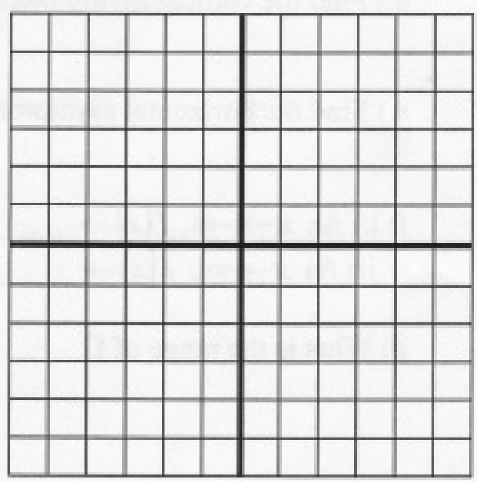
b)

c)

d)

d. Find the intervals of concavity and any inflection points g has.

e) Use all the above parts to help you graph g (you are free to rescale those axes any way you find fitting, but be sure to label)



4.) (15 points) The relationship between the amount of money spent by Cannon Precision Instruments on advertising and Cannon's total sales, S(x) is given by

$$S(x) = -0.002x^3 + 0.6x^2 + x + 500,$$

where ($0 < x < 200$) and x is measured in thousands of dollars.

Use differentials to estimate the change in Cannon's total sales if advertising expenditures are increased from $100,000 to $105,000.

4)

5.) (21 points) Let $f(x) = \dfrac{x^2 - x - 6}{x^2 - 2x - 3}$.

a.) Find the holes of f (if any) and state the domain of f

b.) Find the x-intercept(s) of f (if any).

c.) Find the y-intercept(s) of f (if any).

d.) Find the vertical asymptote(s) of f (if any).

e.) Find the horizontal asymptote of f (if any).

f) i.) As $x \to -\infty$, $f(x) \to$ _______.

ii) As $x \to \infty$, $f(x) \to$ _______.

g) What is the range of f?

5.)
a.)

b.)

c.)

d.)

e.)

f.)

g)

6.) (20 points) Say I want to optimize the planting area in a greenhouse which I am about to build.

Suppose the greenhouse I want has a flat roof and is 6.5feet tall and I have $160ft^2$ of insulating material at my disposal. What are the optimal width and length of the greenhouse?

6)

7). (2 points) Clarity, misuse of notation (other than the limit notation), etc. No response is required here from you, the student.

The following are two bonus problems. They are not part of the regular test but would give you extra points if you do them correctly. Attempt them only if you have extra time.

1^*) Suppose the side of a cube is measured with a maximum percentage error of 2%. Use differentials to estimate the maximum percentage error in the calculated volume.

1^*) %

2^*) Do you see any connection between marginal cost and the concept of differential? Please elaborate.

Math 022 Anguelova

Asymptotes

<u>Definition.</u> The line y = b is a <u>horizontal asymptote</u> of the graph of y = f(x) iff at least one of the following is true: (1) $\lim_{x \to -\infty} f(x) = b$ (2) $\lim_{x \to \infty} f(x) = b$

<u>Definition.</u> The line x = a is a <u>vertical asymptote</u> of the graph of y = f(x) iff at least one of the following is true:

(3) $\lim_{x \to a^-} f(x) = -\infty$ or (4) $\lim_{x \to a^-} f(x) = \infty$ (5) $\lim_{x \to a^+} f(x) = -\infty$ (6) $\lim_{x \to a^+} f(x) = \infty$

<u>Fact.</u> A polynomial function has neither vertical asymptotes nor horizontal asymptotes.

<u>Theorem.</u> For a rational function $r(x) = \frac{N(x)}{D(x)}$ (where N(x) and D(x) are polynomials), if a is a real number for which D(a) = 0 but N(a) ≠0, then the line x = a is a vertical asymptote for the graph of y = f(x).
That is, if a is a zero of the denominator of a rational function, but not a zero of the numerator of the rational function, then the line x = a is definitely a vertical asymptote of the graph.

Notice that if D(a) = 0 but N(a) ≠0, then for the rational function, $\lim_{x \to a} r(x)$ will have the indeterminate form L/0, where L is not zero.

Warning: If both D(a) = 0 and N(a) = 0, then x = a may or may not be a vertical asymptote. One must investigate the limit, as $x \to a$, of r(x).

Example. What can we conclude about vertical asymptotes of $f(x) = (x^2 - 25)/(x^2 - 8x + 15)$?

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} \left(\frac{(x-5)(x+5)}{(x-5)(x-3)} \right) \qquad \lim_{x \to 3} f(x) = \lim_{x \to 3} \left(\frac{(x-5)(x+5)}{(x-5)(x-3)} \right)$$

Guidelines for Sketching the Graph of f

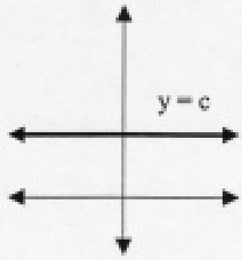
1. Determine the domain of f.
2. Find the x- and y-intercepts of f, if any.
3. Determine the behavior of f(x) as $x \to -\infty$ and as $x \to \infty$.
4. Find any horizontal and/or vertical asymptotes of f.
5. Determine the open intervals on which f is increasing and decreasing.
6. Find the relative extreme values of f, if any, and list the corresponding points on the graph where they occur.
7. Determine the concavity of f, the open intervals on which f is concave upward and concave downward.
8. Find the inflection points of f, if any.
9. Sketch, possibly plotting a *few* additional points that will help show the shape of graph.

Math 22 -- The "Easy" Derivative Rules

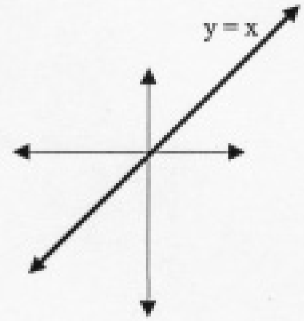
Suppose that c and n are real constants and that $f'(x)$ and $g'(x)$ exist. Then

Derivative of a constant:

$\frac{d}{dx}(c) =$



$\frac{d}{dx}(x) =$



For real constants m and b, $\frac{d}{dx}(mx+b) =$ (What 'shape' does the graph of y = mx + b have?)

Power Rule:.

$\frac{d}{dx}(x^n) =$ provided that the relevant powers of x are real.

Constant Multiple Rule:

$\frac{d}{dx}(cf(x)) =$

Sum Rule:

$\frac{d}{dx}(f(x) \pm g(x)) =$

Examples:

1. If $g(x) = x^4 - \frac{1}{x^4} + 4x - \sqrt[3]{4}$

then g(x) = , so g ' (x) =

2. $\frac{d}{dx}\left(\sqrt[4]{x} - \frac{8}{\sqrt[5]{x}} + \frac{1}{8\sqrt[4]{x}}\right) =$

3. $\frac{d}{dt}\left(\frac{t - 3\sqrt{t} + 12t^5 - 9t^7}{2t^5}\right) =$

Some tips to do well in Math 22

Come to class regularly

When you don't understand something in class ask

Before the next class session read over your notes and make sure you understand them completely. If something is unclear don't hesitate to ask. If you do this you would be prepared to score well on all the quizzes

Keep on top of things and never fall behind on your reading/homework. You should not find yourself cramming for the exams the night before. Mathematics is a subject that needs time to gradually sink in. Once you fall behind it is very hard to catch up.

Make use of my office hours and the math Help center

Do homework regularly and without the aid of the Solutions Manual. Ultimately when you are working on homework you should be able to do the problems without flipping through the book or looking at similar problems we have done and just mimicking the solution.

The more practice problems you do, the better. Doing problems is the only way to practice and gain speed. You would have roughly about 30 problems on the final and 3 hours to complete them, so pace is very essential to do well. Pace comes through lots and lots of practice. The homework I assign is the bare minimum problems to do, so it would be a good practice to do some additional problems in the chapters and problems from old finals.

Learn the formulas

Sometimes it helps to read ahead. This gives you a chance to ask on the spot about the things you did not quite understand or find harder. If you send me an email before class and say this is a hard part, I can try to incorporate it in my lesson plan better and give more examples.

Attend the review sessions before the tests and final and be prepared to ask questions.

Participate in class. I would sometimes give problems to work on independently and then ask you to write your solutions and explain them to the class. Students tend to like those sessions as they give them chance to tackle problems. It is also surprising how lively discussions those sessions arise.

Bring your calculator to class and follow my calculations. It would not only give you practice to make your calculations quickly, but you would be able to catch any calculation mistakes I make.